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CASE STUDY FOR NEW FEATURE EXTRACTION ALGORITHMS, AUTOMATED DATA CLASSIFICATION, AND MODEL-ASSISTED PROBABILITY OF DETECTION EVALUATION (PREPRINT)

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CASE STUDY FOR NEW FEATURE EXTRACTION ALGORITHMS, AUTOMATED DATA CLASSIFICATION, AND MODEL-ASSISTED PROBABILITY OF DETECTION EVALUATION

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Abstract. This paper explores feature extraction algorithms for crack characterization in eddy current inspection of fastener sites. A novel feature extraction method fitting approximate models to data associated with geometric part features addressing adjacent fastener sites and panel edges are developed. Data classification methods in the circumferential direction around fastener sites are developed to better characterize fatigue cracks with improved noise invariance. Model-assisted probability of detection results are presented highlighting the benefit of automation in NDE.

Keywords: eddy current, fastener sites, feature extraction, models, probability of detection (POD)
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INTRODUCTION

The characterization of cracks around fastener sites in multi-layer structures continues to be an important problem for the NDE community. To improve eddy current techniques for this problem, computational methods encompassing numerical models, novel data analysis and classification algorithms, and design optimization tools are proposed with new probe designs. Recently, the development and validation of numerical models based on the volume integral method [1] for the eddy current fastener crack problem have been presented [2]. This design strategy is directed at improving the source field characteristics, measurement sensor design, and data analysis and classification algorithms.

Due to the complexity of aircraft structures, a variety of coherent noise signals can hinder characterization of cracks around fastener sites. For example, although an asymmetric response observed for a fastener site in an eddy current image from 2D raster scan data is typically used to distinguish crack and no crack conditions, there are several potential sources of coherent noise features with similar asymmetric responses. In particular, irregularities in the fastener hole fit condition such as asymmetric gaps between the fastener and hole due to shifted or skewed fasteners, or oblong holes due to poor drilling can produce asymmetric image results. In addition, variability in probe lift-off due to the scanning system hardware alignment or part surface conditions and the irregular quality of windings in eddy current probe can also be sources for measurement asymmetry. Lastly, the presence of adjacent fastener sites, part edges and subsurface structures can also produce localized features around the fastener site and complicate the data interpretation problem. The presence of these examples of coherent noise can increase false call rates and likewise limit crack sizing. Thus, there is a need to develop advanced data analysis approaches and reliable features that are sensitive to the crack condition yet invariant to such coherent noise signals also present in real data. The use of invariant features has been shown to be valuable for other problems in eddy current nondestructive evaluation [3]. In addition, statistical approaches to estimation of image features have been proposed and investigated for NDE applications [4]. The primary objective of this work is to develop model-based feature extraction algorithms and automated data analysis routines for crack characterization with invariance to noise features for eddy current inspection of fastener sites. Two separate feature extraction methods are presented: 1) to address features associated with fatigue cracks in the presence of asymmetric noise at the fastener site and 2) to address fitting approximate models to data associated with geometric part features including adjacent fastener sites and panel edges. Demonstrations of both methods are presented with good results for simulated and experiment data. Promising features indicating the potential for sizing subsurface cracks are also presented.

To validate the performance of new inspection techniques incorporating automated data analysis algorithms, probability of detection (POD) studies are performed including preparation of multiple samples that closely represent the geometry of the structure. These samples include a statistically significant number of inspection opportunities containing a distribution of flaw sizes. The preparation of POD samples and the process to acquire data from these samples is often very time consuming and expensive; thus, model-assisted POD (MAPOD) evaluation methodologies have recently been proposed [5]. Through the use of computer simulations to model the inspection process and statistical methods to determine the POD from a combination of experimental and simulated data, significant cost and time savings can be achieved. However, additional work concerning the validation of models and the MAPOD methodologies is needed to ensure the reliability of this approach. In this work, a MAPOD demonstration study is presented. In addition, the benefit of the automated data analysis algorithms in the MAPOD evaluation process is highlighted.

CIRCUMFERENTIAL FEATURE EXTRACTION FOR CRACK DETECTION

Simulated studies were first used in the demonstration of circumferential feature extraction methods for crack detection around fastener sites. Figure 1(a) presents a plot of the reactance component of impedance with respect to angular location in the circumferential direction with the radial position set to 9 mm. Two levels of probe tilt (0° and 2°,) and three crack sizes (of 0 mm, 1 mm and 2 mm in length) are presented. Of particular interest is a significant localized peak whose magnitude is related to crack length. There is also a second peak due to an increased probe tilt in the reactance data; however, it is much broader with much similarity to a cosine function in profile and period. Although not presented, very similar trends were also observed for the cases of an (a) asymmetric gap between fastener and hole (with an aluminum fastener) and (b) linear liftoff variation (with a titanium fastener) [6].

To extract a measure of this localized crack feature found in the circumferential direction, an approach is proposed through the estimation of the corresponding parameter in a model characteristic function. The feature vector in the circumferential direction can be represented by four model components, (1) a sinusoidal function representing an asymmetric response of a typical non-flaw condition, (2) a Gaussian function representing a localized response associated with a radial crack, (3) constant offset value related to the overall axisymmetric response due to probe, sample geometry and material properties, and (4) random measurement noise. A general form of this relation is given by:

$$f(\theta) = A + \sum_{p=1}^{N_p} B_p \exp\left(-D_p\left(\theta - \varphi_p\right)^2\right) + C\cos\left(\theta + \phi\right) + X(\theta)\,, \tag{1}$$

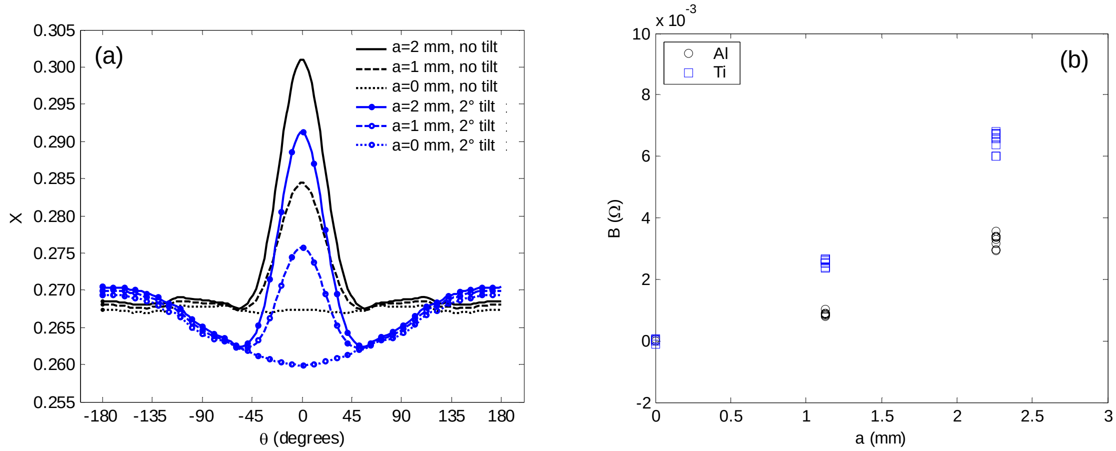

FIGURE 1. Plots of (a) reactance component along circumferential direction at r = 9 mm, for varying probe tilt and flaw size condition (with titanium inserts), (b) Plot of all circumferential crack measure (B_X) results comparing differences in flaw size and insert material type at r = 13 mm [6].

where N_p radial fatigue cracks may be considered with varying magnitude and phase and X, a random variable, is used to represent measurement noise. Conveniently, the summation of the sinusoidal components can be reduced to a single cosine function of magnitude A and phase ϕ, which provides a straightforward means of filtering many asymmetric noise conditions. To evaluate these parameters, a nonlinear least-square estimation approach was used to minimize the error between the characteristic model and the simulated data.

For this simulated study, the model was limited to the presence of a single crack and all phase angles were set equal to zero, given the alignment of the crack, gap asymmetry, probe skew and probe liftoff asymmetry with the x-axis. Constraints were also used on the parameter D, the width of the Gaussian function, to limit the circumferential extent of the peak response. Figure 1(b) presents a comparison of all of the circumferential crack measure (B_X) samples comparing differences in crack size and insert material type for the probe location of r = 13 mm. Of particular interest is that the vast majority of variation is due to the insert material type with very little variability due to the other noise factors. Since the insert material can be found by eddy current measurements made at the center of the fastener, this trend provides a feature for crack sizing with excellent noise invariance.

To validate the proposed feature extraction methodology, a set of experimental data for hidden cracks around fasteners in multilayer structures was analyzed. Details concerning the experimental design and test specimens are presented in prior work [6]. An automated signal classification algorithm was developed based on the proposed circumferential crack feature extraction algorithm with sensitivity to the localize Gaussian response above and below the hole associated with the presence of a crack while rejecting any step changes in the response. The analysis routine was applied to a total of 132 no crack fastener sites and 39 crack cases with lengths ranging from 0.027" to 0.169". When flaws greater than 0.067" are considered and a simple threshold classifier is applied to the circumferential measure, all flaws were detected and a false call rate less than 1% was achieved.

MODEL-BASED FASTENER SITE AND PART EDGE REMOVAL ALGORITHM

Although the proposed circumferential feature extraction methodology is beneficial for distinguishing crack features in the presence of certain asymmetric hole features, liftoff and probe tilt, other complex features of aircraft structures can hinder the direct application of the approach. For the experimental demonstration, all cracks were located perpendicular to the line of fasteners and thus do not require a special means to distinguish them from adjacent fastener sites. For general inspections, the presence of adjacent fastener sites and part edges in close proximity can complicate the search for cracks located 360° around the hole of interest. In addition, they can also contribute to the eddy current response in crack regions and thus impact the estimated magnitude degrading sizing accuracy. There is a clear need for model-based feature extraction schemes to also compensate for adjacent fastener sites and part edges.

To both accurately and efficiently represent fastener sites and part edges in eddy current measurement models, estimation using approximate representations was investigated. A collection of voltage measurements $V_{i,j}$ are considered at $[x_i,y_i]$ where $i = 1,2,\ldots N_i$ data samples for the two measurement components ($j = 1,2$) representing the real (in-phase) and imaginary (quadrature) parts. The measurement model can be approximated by

$$V_{i,j} = R_{i,j} + \sum_{k=1}^{N_k} S_{i,j,k} + E_{i,j}\,, \tag{2}$$

where $R_{i,j}$ represents the response due to an unflawed specimen (without fastener sites), $S_{i,j}$ represents the response for each fastener site $k = 1,2,\ldots N_k$ including all cracks and asymmetric hole features, and $E_{i,j}$ represents noise due to measurement error ($e_{i,j}$) and coherent noise due to systematic model error ($\varepsilon_{i,j}$).

The fastener site measurement model is first considered. The proposed model including a single fatigue crack can be decomposed into axisymmetric ($g_{j,k}$) and higher order components:

$$S_{i,j,k} = g_{j,k}(r_{i,k}) + B_{j,k}(r_{i,k})\exp\left(-D_{j,k}\left(\theta_{i,k} - \varphi_{j,k}\right)^2\right) + C_{j,k}(r_{i,k})\cos\left(\theta_{i,k} + \phi_{j,k}\right) + X_{i,j,k} \tag{3}$$

where each fastener site is centered at $[x_{0,k},y_{0,k}]$, and $r_{i,k} = \sqrt{\left(x_i - x_{0,k}\right)^2 + \left(y_i - y_{0,k}\right)^2}$, $\theta_{i,k} = \tan^{-1}\left[\left(y_i - y_{0,k}\right)/\left(x_i - x_{0,k}\right)\right]$. To efficiently and accurately represent the axisymmetric response at a general fastener site as shown in Figure 2, a two part function is used to decompose the measurement data

$$g_{j,k}(r_{i,k}) = \begin{cases} \sum_{l=0}^{N_l} A_{j,k,l}\, r_{i,k}^{l} & r_{i,k} \le r_b \\ G_{j,k}(A_{j,k,l})\exp\left[-\beta_{j,k}(A_{j,k,l})\, r_{i,k}^{l}\right] & r_{i,k} > r_b \end{cases}. \tag{4}$$

A polynomial representation was found to ideally represent the primary response and subtle differences in fastener sites using few parameters and minimal function evaluation time. The number of polynomial terms, N_l, of 10 was found to produce good results. To ensure the response converges to zero as $r_{i,k}$ becomes large, an exponential decay term was added. Continuity of the expression for all $r_{i,k}$ is maintained through the fitting of the exponential decay term (in the neighborhood of r_b) using the results of the polynomial fit, $A_{j,k,l}$. For the steel fasteners present with titanium fasteners, the saturated signal found at the center of fastener response can be addressed using an additional region represented by a polynomial.

For the first order components of the fastener fit problem, the solution vector fastener center and axisymmetric response is given by $\mathbf{z} = [x_{0,k},\ y_{0,k},\ A_{j,k,l}]$, for j measurement components, k fastener sites and l polynomial terms. Additional unknowns for the part (edge), crack and noise terms must also be included to achieve good results for the global error minimization problem. Since the global solution for all of these unknowns cannot be readily solved as a large single error minimization problem quickly and reliably, the new

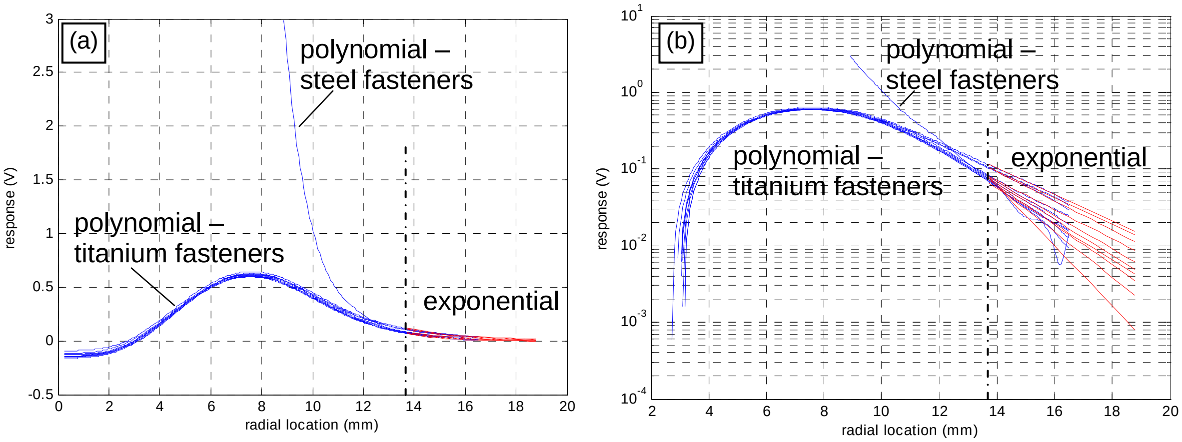

FIGURE 2. Plots of experimental axisymmetric fastener site responses in (a) linear and (b) log scales.

solution strategy presented here focuses on three steps: 1) a heuristic approach using a physical understanding of the sources of greatest error, 2) a least-square estimation approach to solving for the polynomial response quickly and accurately, and 3) an iterative approach to improve model solutions for overlapping fastener site and part edge regions.

The parameter estimation approach was designed to simplify the global estimation problem into a series of localized estimation problems combined with iterative schemes to address coupling between the parameters across all of the data. The first task is to estimate the location of the fastener site centers. Figure 3(a) displays an image plot of the original measurement data for the in-phase component (V_x) containing 10 fastener sites (9 of titanium, 1 of steel). To achieve precise hole removal, evaluation of the fastener site center is performed at a resolution smaller than the scan step size. Correlation methods using iterations of varying resolution in 2D were used to calculate the initial hole centers. Fastener site material type is then evaluated based on the measurement response at each hole center. Both the fastener site and crack characteristic response will vary with fastener material type. Next, data in regions far from the hole center but near the panel edge is used to estimate the characteristic response due to the panel edge. Figure 3(b) displays an image plot of the processed data with hole feature extraction, presenting the characteristic response produced as the probe approaches the panel edges in the top and bottom. Expressions for the measurement response model at the panel edges in the y-direction are given by

$$R_{i,j} = \alpha_{0,j} + \beta_{0,j} \exp\left[-\beta_{1,j}\left(y_i - \beta_{2,j} - \beta_{3,j}x_i\right)^2\right] + \gamma_{0,j} \exp\left[-\gamma_{1,j}\left(y_i - \gamma_{2,j} - \gamma_{3,j}x_i\right)^2\right]. \quad (5)$$

The tail of a Gaussian function is used to represent the decay of the edge response as the probe is moved toward the center of the part. Also, cross terms between the x and y directions are included to address varying alignment between part edges and scan directions.

Next, estimation of the axisymmetric fastener site model parameters ($A_{j,k,l}$) are evaluated in conjunction with an heuristic iterative approach. Local regions are defined around each fastener site center to estimate the axisymmetric model parameters. The order of the fastener site evaluations is dependent upon fastener material type and location in the image data. Steel fasteners produce significantly larger eddy current responses with respect to titanium and aluminum fasteners and typically result in greater error between the model and actual data. Thus, the iterative process begins with these fastener sites for model estimation. Adjacent fasteners to steel fasteners, non-end fasteners and end fasteners are

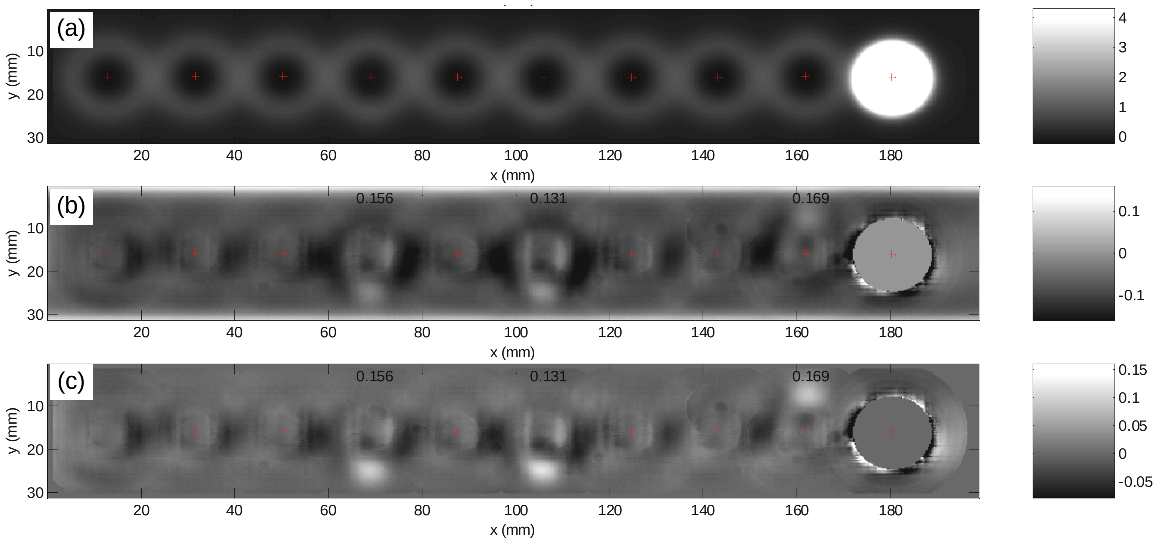

FIGURE 3. Image plots of (a) original measurement data for in-phase component (V_x) and corresponding (b) processed data with hole feature extraction and (c) processed data with hole and edge feature extraction.

then subsequently addressed according to differing levels of potential model error. The least square error estimation is thus performed in an iterative sense for both the polynomial model representation ($A_{j,k,l}$) and also the fastener center [$x_{0,k}$,$y_{0,k}$] and their convergence is evaluated. Figure 3(c) displays an image plot of the processed data with both hole and part edge feature extraction. For this specimen with three cracks located around fastener sites, the crack features are more clearly observed. In addition, other weak features associated with the surface condition (producing localized changes in liftoff due to bumps or missing paint) were also observed. A complete automated process performs this feature extraction algorithm in approximate 60 sec for a 10 hole panel, providing far greater accuracy and a 10X improvement in speed over prior experience with direct global estimation methods.

Two additional refinements to the estimation problem are proposed to address error in the axisymmetric fastener site model between two adjacent fasteners and the presence of cracks in the measurement data. First, the approximate model proposed for representation of fastener sites assumes that there is no significant interaction between the eddy currents generated between adjacent fasteners and thus no error in the superposition of adjacent fastener site models. To validate the accuracy of this assumption for the problem, simulated studies in VIC-3D® were performed. Numerical simulations were performed for a single fastener site and two fastener sites separated by 0.75" (19 mm) as shown in Figure 4(a). The single fastener site result was applied to both fastener site locations and the data was superimposed to compare results with the exact numerical case for the two adjacent fastener. Figure 4(b) shows an image plot of the percent error between the superposition of axisymmetric fastener site models with respect to full numerical solution. The peak error of 2.1% can be found at a point equidistant between the two fastener centers. This adjacent fastener superposition error corresponds well with the remaining error observed between fastener sites in the experimental data shown in Figure 3(c). Initially, error weighing in this region is used to improve the estimation of the fastener site center and the polynomial representation. Inclusion and estimation of the adjacent fastener superposition error in the model is recommended for future work. Large crack signals can also impact both proper centering and fastener model parameter estimation. Upon detection of the crack condition, error weighing in the neighborhood of the crack was also used in this study. For future work, the inclusion of a crack model in the estimation problem is recommended.

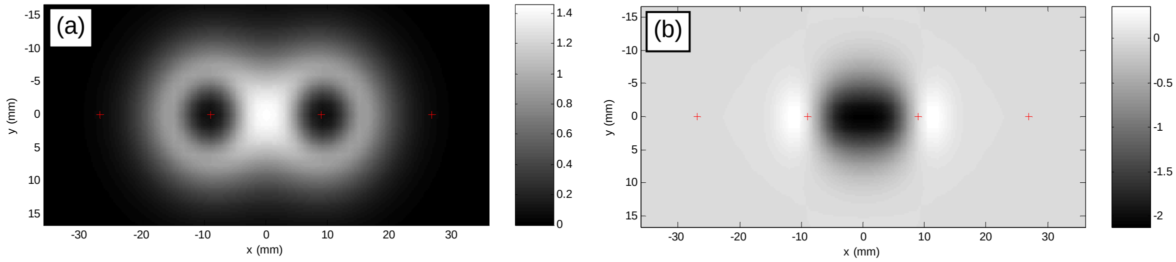

FIGURE 4. Image plots of (a) simulated results for two adjacent titanium fastener sites and (b) percent error between between superposition of axisymmetric fastener site models with respect to full numerical solution.

DISCUSSION OF MODEL-ASSISTED POD EVALUATION AND AUTOMATED DATA ANALYSIS

A demonstration is also presented for a full model-assisted (FMA) POD methodology incorporating computer simulation for the inspection of cracks around fastener sites in a two layer aircraft wing-type structure inspection performed with an eddy current technique. To validate the proposed model-assisted POD methodology, a set of experimental data for hidden cracks around fasteners in multilayer structures was analyzed [6]. Using methods and experimental data presented in the prior sections, the new feature extraction algorithms and automated signal classification routines were successfully applied to the experimental data set demonstrating the ability to detect small cracks around fasteners while maintaining a low false call rate. VIC-3D®, a commercial software package based on the volume integral method [1], was used to perform the simulations and found to address a great majority of the model variables. Simulated parametric studies were performed for varying crack size and type, considering both through and corner crack geometries. The simulated and experimental data for a given probe and inspection geometry were equated using a calibration procedure developed in prior work [6]. Figure 5(a) presents a comparison of the experimental and simulated data for varying crack length and crack type. In general, there is good agreement between the two data sets with some error for select experimental data points. A Monte Carlo simulation was performed using the noise distribution for the unflawed fastener sites, the corner crack model, and the through crack model, to populate the full-model assisted data sets for POD evaluation. Additional details on the FMA methodology applied to this problem can be found in [7].

Figure 5(b) presents a comparison of the POD results for the experimental and full-model assisted approaches. A good match was achieved between experimental and full-model assisted (FMA) approaches where the FMA POD curve was found to be within the

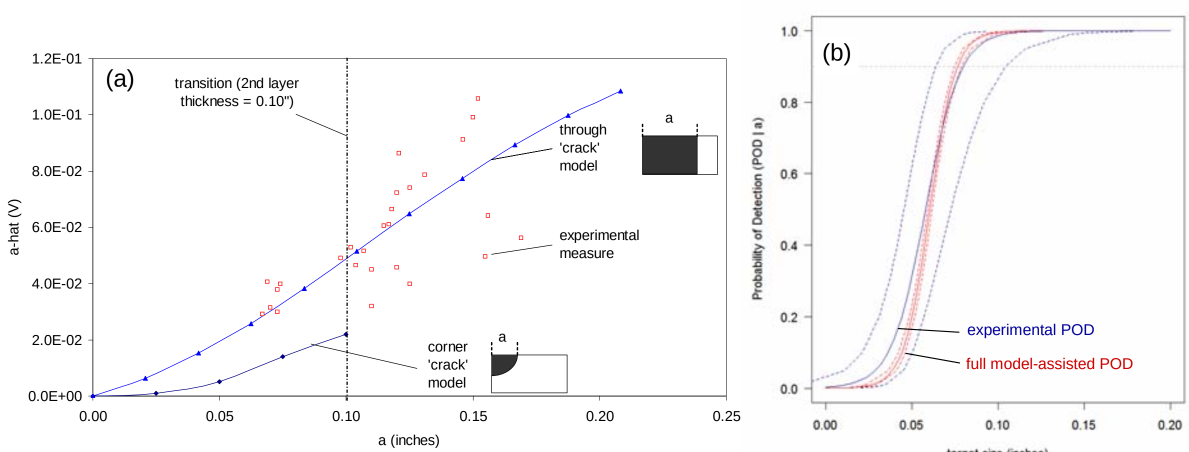

FIGURE 5. (a) Comparison of experimental and simulated data for varying crack length and type, and (b) corresponding POD evaluation results for experimental and full model-assisted POD studies.

confidence bounds (95%) of the empirical POD curve. From this study, automation was found to play a key role in the success of the model-assisted evaluation of the NDE technique. First, critical NDE system parameters can better controlled using automated scanning systems. In particular, there is less opportunity for variation related to human operation of equipment, which can be significant given the variation in skill and experience in inspectors. Second, the proper application of automated signal classification algorithms has the potential to also address sensitivity to certain NDE measurement and part factor variation. In particular, algorithms processing noise invariant features in the eddy current data as observed in the circumferential direction can be used to minimize sensitivity to asymmetric hole response features and lift-off for the fastener crack problem. Applying greater control and decreasing sensitivity to NDE system parameters results in greater success of MAPOD methods to accurately model the NDE measurement signals.

CONCLUSIONS AND RECOMMENDATIONS

New model-based feature extraction methods were presented to improve sensitivity to crack features in data with coherent noise signals while also improving processing time. First, data analysis methods in the circumferential direction around the fastener hole were developed to better characterize fatigue cracks with improved noise invariance. Second, a novel feature extraction method fitting approximate models to data associated with geometric part features such as adjacent fastener sites and panel edges was developed and demonstrated. Model-assisted probability of detection results are also presented highlighting the benefit of the automated signal classification algorithms. Good agreement was found between the experimental and full-model assisted POD evaluation results. Future work is planned to further refine the algorithms for improve crack characterization and advanced probe designs incorporating GMR sensors. Significant challenges also remain in developing model-assisted POD procedures, including improvements to the existing computer modeling software, optimization of method to combine both test samples and computer simulations, and determination of the uncertainty in the evaluation process.

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